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Teaching Mathematics: Dialogues Between Situated Learning Perspective and Wittgenstein's Language Games

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Abstract: This paper fosters a dialogue between L. Wittgenstein's concept of "language games" and J. Lave and E. Wenger's theory of "situated learning." It posits that mathematics is not a discovery, but a invention, a culturally embedded practice shaped by social interactions and contextual influences. The discussion emphasizes that mathematical understanding is deeply rooted in the activities through which mathematical objects are created and explored, viewing these objects as cultural artifacts. The article advocates for an educational paradigm that recognizes the contextual and socially situated nature of mathematics, thereby democratizing access to mathematical knowledge and challenging entrenched hierarchies within mathematical practices.

Keywords: Language games; Situated learning; Mathematics Education.
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Introduction

We perceive and engage with the world through a diverse array of sign systems or semiotic resources, such as language, visual images, sounds, and gestures. These resources are employed by social groups within specific historical and cultural contexts to construct and define both culture and reality (Chouliaraki, 2008). Mathematics, as a uniquely human endeavor, represents one such mode of interaction with the world. Unlike tangible objects encountered in everyday life, mathematical objects are distinct in that they do not present themselves directly. Instead, they arise through mathematical activity as cultural artifacts. These objects exist as representations mediated by signs, with their meanings unfolding through the development of structural and logical frameworks to which they conform. Consequently, mathematical objects are intrinsically linked to the activities that generate and explore them, presenting themselves as conceptual challenges to human understanding (Otte, 2005).

> Mathematical objects are at first nothing but objects of activity (e.g., problems) represented by indexical signs whose meaning unfolds in the elaboration of the structural and lawful determinations to which they are subject. Insofar, whereas mathematical objects are given to activity, they are "given as tasks" to understanding (Otte, 2005, p.11).

Engaging with mathematical objects requires recognizing their unique characteristics and connecting them to specific practices. This process involves activating various skills, such as thinking mathematically, representing and manipulating these objects, clearly expressing ideas, reasoning and communicating from a mathematical perspective, solving and formulating problems, modeling real-world situations, and utilizing technological tools to support these objectives.

Notably, such perspectives bring mathematics closer to the realm of human endeavor. However, this view is not the only conceptualization of mathematics. In contrast, the Platonic perspective has dominated mathematical discourse for centuries and continues to exert significant influence. According to Platonist thought, the abstract nature of mathematics situates it outside the human domain, placing it within the realm of ideality. Achieving mathematical understanding, from this perspective, requires recalling pure or perfect ideas once contemplated by the soul in a prior existence.

This article seeks to foster a dialogue between two perspectives that aim to bring mathematics closer to human practices: L. Wittgenstein's concept of "language games" (Wittgenstein, 2009) and J. Lave and E. Wenger's theory of "situated learning"(Lave & Wenger, 1991).

Wittgenstein's language games: preliminary thoughts

Wittgenstein was born in Vienna in 1889. His childhood and youth were shaped by a family environment that fostered study and encouraged the development of critical and artistic thinking. During the First World War, Wittgenstein enlisted voluntarily. Amid his travels to and from the front, he developed his first and most significant work, the Tractatus Logico-Philosophicus, which was published a few years after the war. In this book, Wittgenstein seeks to clarify the logical conditions required for thought and language to represent the world. It is widely believed that his wartime experiences brought his studies in logic into closer alignment with ethical and religious themes.

> Part of the difficulty, and the charm of the book, lies in the fact that it discusses problems such as linguistic meaning, the nature of logic, the aim of philosophy and the place of the self, in a way that combines the formal and the romantic (Glock, 1998, p.356).

The work is divided into four parts: the theory of logic, the pictorial theory, a discussion on science and mathematics, and a discussion on the mystical. Its propositions are ontological in nature, explaining mathematics as a dimension of logical operations and presenting language through a more realistic perspective.

According to Glock (1998), Wittgenstein draws upon certain Fregean conceptions. However, for Wittgenstein, the meaning of a proposition is neither the object to which it corresponds nor the manner in which a truth value is presented. Instead, it is primarily a possibility—a potential combination of objects that does not necessarily need to be realized. In this sense, meaning precedes facts. The meaning of an elementary proposition is determined by the meanings of its simplest components.

After the publication of the Tractatus, from the late 1920s to the mid-1940s, the Philosophy of Mathematics has occupied Wittgenstein's philosophical thoughts since. The work of this period, marked by his conception of calculus, reveals the repositioning of the world of formal logic and precision of meaning identified in the Tractatus Logico-

Philosophicus. In this period, Wittgenstein tries to counter the view that mathematical truths are somehow transcendental: a mathematical proposition has truth and meaning independent of human rules or their use. This view implies two issues: there is a mathematical reality independent of practice and that it is the task of mathematicians to discover this reality.

The primary criticism of this conception is that it positions mathematical reality as a guarantor of the precision of mathematics, resulting in what Wittgenstein describes as a misleading image of the nature of mathematics (Wittgenstein, 2009). To illustrate this point, Wittgenstein references a passage from Godfrey Hardy's 1940 book In Defense of a Mathematician: "317 is a prime number, not because we think it is, or because our minds are formed in this or that way, but because it is, because mathematical reality is constructed in this way" (Hardy, 2000, p.24).

Given the rules of number theory, the statement about number 317 is indeed true. However, the central question concerns the reality to which this proposition corresponds. In this regard, Wittgenstein suggested that such propositions may bear a relationship of responsibility to this reality. This idea can be explored in two ways: an internal perspective, in which each mathematical proposition relates to another within the system, and an external perspective, in which the entire body of mathematics corresponds to something larger. It is one thing to say that "12 - 7 = 4" is incorrect within our numerical and arithmetic system; it is quite another to claim that the entire system is flawed because it fails to perfectly represent mathematical reality.

The second criticism of the hardyan image arises from the claim that the mathematician's role is to discover mathematical reality. According to Wittgenstein, treating mathematics as the discovery of pre-existing object blurs the distinction between mathematics and the empirical sciences. Wittgenstein contends that mathematics should be understood as an invention rather than a discovery. However, while a child learning the number system is engaging in the discovery of mathematics, they are not inventing it. In contrast, Wittgenstein focuses on the professional mathematician, who invents new concepts and principles.

The objects studied by mathematicians differ fundamentally from those studied by the empirical sciences. Mathematical objects are precise and can only be accessed and manipulated through representation. Consequently, mathematical language and empirical-descriptive language differ in two keyways: first, the criterion of truth in mathematics is inherently precise; second, precision itself is an essential and defining feature of mathematical language.

Some sensory impressions can mislead us when observing phenomena or even lead to the revision of conclusions drawn from previous observations. In mathematics, however, sensory impressions are irrelevant to the truth of a proposition. For instance, the way we use the statement "2 + 2 = 4" and the role it plays in our lives is fundamentally different from the statement "it's raining today." According to Wittgenstein, the former is a necessary proposition, whereas the latter is contingent (Wittgenstein, 2009; Gerrard, 1987).

Wittgenstein also emphasized that the most important characteristic of mathematics is the non-revisability of mathematical sentences (Gerrard, 1991). However, it is important to note that these mathematical sentences assume their role within a particular context and for a specific period. There is no guarantee that mathematical sentences will remain unchanged indefinitely; they may evolve when necessary. For instance, the numerical value of the expression "2 - 4" was meaningless until the introduction of negative numbers. The concept of non-revisability can be more precisely understood as the idea that no sensory or empirical impression can alter a mathematical sentence. Mathematics is "non-revisable" because it fulfills a role similar to that of a standard meter bar: it serves as the measure, not the object being measured. In this sense, mathematics assumes a normative function.

From his critique of the hardyan image, Wittgenstein developed his conception of calculus, which is characterized by the following principles: i. each calculus is a closed and autonomous system, immune to external criticism; ii. there is a clear distinction between a calculation and its practical applications; iii. any revision to its rules, no matter how minor, results in a completely new calculus; iv. discussions can only address individual calculations, as there is no possibility of considering calculations in general (Gerrard, 1987, 1991).

According to Gerrard (1991), two philosophies of mathematics emerge during this period, both shaped by Wittgenstein's opposition to the hardyan image. One is rooted in the principles of his calculus conception, while the other leans toward the idea of language games. Notably, as each of the above principles is questioned, the concept of language games becomes increasingly evident.

This shift in perspective and the subsequent revision of principles stem from Wittgenstein's studies on contradictions. In 1901, Bertrand Russell introduced his now-famous paradox—commonly referred to as the Barber Paradox—in which he demonstrated that "there is no set of all sets." This discovery had significant repercussions, particularly for Gotlob Frege, who was in the process of completing the second volume of his foundational work. Frege was compelled to include an appendix acknowledging the paradox, a development that not only disrupted his work but also unsettled the entire mathematical community. Wittgenstein's perspective on contradictions is particularly noteworthy:

> I want to oppose the bogeyman of contradiction, this superstitious fear that leads the discovery of a contradiction to mean the destruction of the calculus. Someone might say: Russell's contradiction has put us on our guard. A contradiction can arise from anywhere. To this, we can say: don't be so nervous. You're being stupid (Gerrard, 1987, p.55).

Wittgenstein's argument, directed at mathematicians, is that if a calculation is flawed due to a contradiction, the appropriate response is to correct it. However, he emphasizes that mathematicians must recognize they are adapting rules for their own purposes, rather than adhering to some mysterious, pre-existing mathematical reality. Alternatively, he suggests that a calculation or problem generated by a contradiction can simply be abandoned if it proves unworkable. Wittgenstein sought to convey that contradictions are not inherently destructive, but that their harmfulness depends on context. For instance, consider the contradiction: "bring me the pencil and don't bring the pencil." A person hearing such a statement might either be confused or amused. Here, the function of the contradiction could be to provoke laughter or confusion. In this context, the contradiction achieves its purpose and is not harmful. Thus, there is no intrinsic reason to avoid a contradiction merely because it exists.

However, there are contexts where contradictions must indeed be avoided. For example, if an engineer designing a bridge employs two calculation methods and arrives at substantially different results, the contradiction must be resolved to ensure safety and precision. In such cases, the consequences of a contradiction can be genuinely harmful, and it is crucial to specify exactly where the issue lies.

This succession of cases serves to illustrate Wittgenstein's point that contradictions are not inherently harmful; they become harmful only in relation to a specific purpose. Put differently, a contradiction is not harmful a priori but only when it results in harm. To rephrase, it is only in the practical use or application of a calculation containing a contradiction that the contradiction becomes problematic (Gerrard, 1987). Gerrard (1987) further notes that this perspective leads Wittgenstein toward a more "mature" philosophy of language. “Our grammar is responsible for the rules we ourselves create, not for a linguistic reality existing only in the sky. If we make mistakes in constructing these rules, it is the use of language that will bring us down, not some kind of a priori grammarian” (Gerrard, 1987, p.61). This view underscores Wittgenstein's belief that the rules governing language and mathematics are human constructs, accountable only to their practical function and use, rather than to some external, metaphysical linguistic reality.

Throughout Wittgenstein's studies and discussions on contradictions, the principles underlying his conception of calculus are systematically challenged. The most immediate change concerns principle “ii. there is a clear separation between the calculus and its practical use.” Wittgenstein came to understand, as a result of his reflections on contradictions, that language is inseparable from the activities and contexts in which it is used. Consequently, mathematics can no longer be understood as consisting of isolated calculations detached from everything else. Mathematical language is, instead, part of a broader linguistic framework.

This shift in principle “ii” directly impacts principles “iii. any revision to the rules, no matter how small, implies a completely new calculation” and “iv. it is only possible to discuss each calculation individually.” The isolation of individual parts of mathematics, as required by these principles, can no longer be sustained. Together, these principles created rigid boundaries between different calculations and prohibited the imposition of overarching general conditions on all calculations. However, if rules derive their meaning from their practical context, calculations can no longer be viewed in isolation. Mathematics, for Wittgenstein, transforms from a closed system to a language game. Principle “i. each calculus is a closed and autonomous system, exempt from any external criticism” is not discarded entirely but is reinterpreted. Wittgenstein argues that anything outside the context of a language game is meaningless. A mathematical symbol has meaning only within the language game of mathematics itself.

By engaging with paradoxes and contradictions, Wittgenstein established a profound connection between mathematics and language games. In a sense, the evolution of Wittgenstein's work on contradictions parallels his intellectual journey toward Philosophical Investigations, where the notion of language games is central. The choice of the term "contemplates" rather than "defines" is deliberate here, as in Philosophical Investigations, Wittgenstein avoids creating rigid definitions. Instead, he offers numerous examples and situations to analyze the varied uses of a particular activity within different contexts. He refers to this method as philosophical therapy.

The goal of philosophical therapy is not to establish the foundations of mathematics or language but to employ grammatical descriptions to dissolve philosophical confusions and challenge exclusivist standpoints. According to Vilela (2013) this approach:

> focuses on enunciations or privileged images associated with them, which point to a referential representation of mathematics. (...) It seeks to undo the image of mathematics as the queen of the sciences which, due to its deductive method, seems to produce absolute truths, and which imprisons us within this logical game and would prevent us from seeing other mathematics (p.34).

Thus, Wittgenstein's philosophical therapy seeks to liberate mathematics from its perceived infallibility and open the way to a more nuanced, context-sensitive understanding of mathematical practices.

Mathematics as language games

In Philosophical Investigations, published in 1953, Wittgenstein's engagement with Augustine's description of language learning underscores his broader critique of oversimplified conceptions of language. Augustine's perspective on language reflects his nuanced understanding of its dual nature: both as a vital tool for communication and as inherently limited in expressing ultimate truths, particularly divine or spiritual realities. He underscores the symbolic and referential nature of words, emphasizing that language does not embody the essence of what it signifies but rather serves as a guide to deeper truths. This aligns with his theological framework, where the material world—including language—is subordinate to the immaterial, eternal truths of God. For Augustine, language facilitates thought and community, yet it also serves as a reminder of the human condition's limitations, pointing toward the ineffable divine that lies beyond human comprehension (Augustine, 1973).

Augustine's image, as Wittgenstein (2009) frames it, depicts language as a system primarily rooted in reference — the association of words with objects, gestures, and experiences through observation and repetition. While Wittgenstein acknowledges this as a plausible and useful depiction of one aspect of language, he resists the idea of treating it as a comprehensive explanation of language's structure and function. For Wittgenstein, the Augustinian image is an example of how a specific linguistic scenario can be misleadingly expanded into a general theory. The key flaw, according to Wittgenstein, lies in ignoring the practical, rule-governed, and socially embedded aspects of language use. He asserts that meaning emerges not solely through naming or reference, but through the activities and contexts in which words are employed — what he later describes as language games. This emphasis on use challenges the Augustinian image, which presupposes that language learning and understanding occur primarily through an isolated process of correlating words with objects.

Wittgenstein's critique can also be connected to his transitional phase, as Gerrard (1987) suggests, where he scrutinized the hardyan image — another philosophical construct characterized by an overgeneralized interpretation of language. Both critiques reflect Wittgenstein's method of dismantling "pictures" of language that philosophers are tempted to treat as explanatory theories. For Wittgenstein, such images obscure the complexity and multiplicity of linguistic practices.

Thus, the critique of the Augustinian image becomes a central thread in Philosophical Investigations, representing Wittgenstein's broader project of showing how misleading images, when uncritically generalized, create conceptual confusions about the nature of language.

To support this idea, Wittgenstein (2009) proposes the following scenario: a builder, referred to as 'A' is using various building stones — blocks, pillars, slabs, and beams — to construct a wall. His assistant, 'B', is tasked with passing the stones in the order in which A requires them. When builder A shouts, "Block!", assistant B hands him a block. It is important to note that when A gives the order, he is not explicitly teaching the helper that the object is called a "block." Within this process of word usage, or what Wittgenstein refers to as a language game, both participants understand the order "Block!" to mean something akin to "Please pass me a block." However, at some earlier point, these characters likely engaged in a preparatory phase, during which the word "block" was introduced with a referential function. In this phase, the name "block" may have been established through an ostensive gesture.

According to Gottschalk (2004), an ostensive definition differs from an explanation because defining is constitutive—meaning it "inaugurates the object"—whereas explaining is descriptive. Similarly, naming serves merely as a preparatory step for describing, functioning as part of a preliminary language game. Ostensive definitions involve explaining the meaning of a word through a combination of a demonstrative expression, such as "This is...," a gesture or point, and the object being referenced. For instance, one might say, "This is white," while pointing to a white object. However, this process assumes that participants in the language game are familiar with a "grammar of colors." Without such familiarity, any other characteristic of the object could mistakenly be interpreted as "white." For example, if the object in question were a "white vase," someone unfamiliar with color grammar might interpret "white" as referring to the vase itself or even its shape. In this context, it is crucial to understand that the object being pointed to provides an example of the word's name ("white") rather than its full meaning.

The ostensive gesture functions as a linguistic tool that establishes an internal connection between a word and the object it designates. However, one can imagine a different "form of life" in which the same gesture holds an entirely different meaning. According to Wittgenstein, the meaning of such a gesture is determined by the specific language game within which it operates (Gottschalk, 2004).

Within the language game described, and assuming knowledge of a grammar of colors, the designation of a single object, such as a “white vase,” to define the term “white” may prove insufficient. How, for example, can one distinguish between white and light gray? Or differentiate between dark blue and light blue while still recognizing both as “blue”? When the term “white” is defined with reference to the “vase,” it becomes a paradigm for “white.”

This paradigm functions similarly to the standard meter bar, which defines the concept of a “meter.” It establishes a standard for what is considered “white.” For an object to serve as a paradigm, it must be considered within the context of its organizational framework. Its paradigmatic status reflects the regulated system of objects to which it belongs, enabling it to function as a “means of presentation” or a norm. For instance, an object can serve as a sample of a color, a standard of measurement, or a model of a word, but only if it has been systematically organized according to a theory or technique — whether of colors, measurements, or vocabulary. As Moreno (1993, p.19) explains, “the paradigm is an agreed rule, not a data”.

From the forementioned paradigm of ‘white”, one can infer the various shades that may still be categorized as “white” and determine the point at which they transition to “gray” based on similarities in color and tone across objects. The idea of differentiating between shades of white may initially seem counterintuitive. For most people, there is only “one white.” However, in Inuit culture, more than 150 shades of white are distinguished due to the specific conditions of their environment and way of life. This suggests that paradigms are closely tied to the cultural and practical contexts in which they arise.

This process reflects what Wittgenstein describes as “seeing something common” or identifying “family resemblances” (Wittgenstein, 2009). Wittgenstein's notion of family resemblance serves as a significant counterpoint to essentialism by rejecting the idea that concepts must be defined by a single, unchanging essence. Instead, Wittgenstein suggests that what unites instances of a concept is not a singular defining feature but a "complex network of similarities" that crisscross and intersect. This idea is exemplified in the analogy of family members: they do not share one identical trait, but rather a series of overlapping characteristics, such as facial features, mannerisms, or voice. Similarly, the concept of a "rope" illustrates this point—its strength and unity derive not from one continuous thread but from the overlapping and interwoven strands that together form a cohesive whole (Glock, 1998).

This perspective challenges essentialist thinking by emphasizing fluidity, variation, and context-dependence in the way concepts function. It highlights the dynamic, multifaceted nature of language and categorization in Wittgenstein's later philosophy.

In the example of the builder and his assistant, Wittgenstein's concepts of "form of life," "family resemblance," and "language games" come together to highlight the situational and contextual nature of language. The shared "form of life" between the builder and assistant enables them to mutually understand and act upon the word "Block!" without requiring an explicit definition of the term. Their common participation in the language game provides a framework of rules—though these rules are not rigid or essentialist, but rather grounded in practice and shared activity.

For a third party unfamiliar with the language game, "Block!" might evoke entirely different interpretations, depending on their assumptions, experiences, or lack of exposure to the game's context. This illustrates Wittgenstein's point that meaning is not inherent in a word but arises from its use within a particular practice. The established understanding between the builder and assistant — anchored in the rules of the game — highlights how words acquire meaning through activity, not through abstract definitions.

According to Glock (1998) rules function as standards of correctness within a language game. They do not describe actions but rather set the conditions for what counts as acting appropriately or sensibly. For Wittgenstein, learning a language game is akin to learning to follow its rules, which are embedded in the shared practices and context of a "form of life." Gottschalk (2004) emphasizes this point for education: teaching words in isolation from practical applications is ineffective. Words gain meaning only through use, as learners internalize the rules of a given linguistic context.

Wittgenstein, therefore, regards mathematics as a language game, though not just any language game. One of its most significant characteristics is the non-revisability of mathematical propositions. However, this does not imply that mathematics is static. Historically, certain propositions were not initially understood or deemed necessary. For instance, at one point in history, the proposition “$2 - 4 = -2$” was nonsensical until circumstances arose that necessitated its acceptance. It is also crucial to clarify that non-revisability applies to empiricism. In other words, mathematical propositions cannot be revised through experimental methods. The validity and meaning of mathematical expressions derive from pre-established mathematical propositions, which serve as institutionalized conventions within a given community (Gottschalk, 2004).

Dialogues with situated learning perspective

In the same way that the concept of language games brings mathematics closer to its practical uses, we need to explore teaching and learning possibilities that address knowledge production from this perspective. In this context, the notions of situated learning and communities of practice, as proposed by Lave & Wenger (1991), will be examined.

Situated theories, developed and consolidated from the late 1980s onwards, introduced significantly different perspectives on the concepts of knowledge and learning. Drawing on ethnographic research conducted in Liberia during the 1970s, Lave (1988) analyzed how apprentice tailors learned their trade. Her findings challenged schoolcentered perspectives on the learning process, raising profound questions about how learning occurs outside formal educational settings.

According to Lave (1996), every activity is embedded in the relationships among people, contexts, and practices. This implies that learning is also situated within complex communities of practice. These communities, as defined by Wenger (1998), consist of groups of individuals engaged in a collective learning process within a shared domain of human endeavor. They are united by a common concern for a specific practice. Wenger (1998) identifies three critical elements that distinguish a community of practice from other types of groups: i. domain: a community of practice has an identity defined by a shared domain of interest. This involves a commitment and shared competence that differentiates its members from others. It is more than a network of connections or a group of friends; ii. community: members engage in joint activities, discussions, and mutual support while sharing information and building relationships that foster collective learning; iii. practice: the community develops a shared repertoire of resources, including experiences, stories, tools, and methods for addressing recurring problems. This requires sustained interaction and engagement over time.

In a community of practice, learning is not shaped by naturalized assumptions about knowledge acquisition. Instead, learning is embedded within practice and is integral to social interaction. In this sense, learners are not passive recipients of knowledge from an expert. Instead, they engage collaboratively with peers and mentors in an active, multifaceted, and sometimes contradictory learning process. Wenger (1998) suggests that learning is better understood as the evolution of participation in ongoing practices rather than through naturalized assumptions about knowledge acquisition. From this perspective, learning is not merely situated in practice but is an integral part of the social fabric of the world in which we live. It is a process by which individuals gradually become members of a community of practice. This progression often begins with "legitimate peripheral participation" (Wenger, 1998; Lave & Wenger, 1991), where newcomers observe the practices and boundaries of the community. Furthermore, Lave and Wenger (1998) highlight that learning through legitimate peripheral participation can occur regardless of whether there is an intentional teaching context. As learners deepen their participation, they transition from peripheral observers to active participants or full agents. It is through this process that newcomers progressively appropriate the cultural repertoire of the group and eventually become full members of the community. As their participation and learning deepen, they transition from peripheral observers to full participants. For this to happen, however, newcomers must have broad access to mature arenas of practice where they can meaningfully engage.

By aligning Lave and Wenger's theories with Wittgenstein's perspective, a community of practice can be seen as a "form of life" that employs specific language games. For Wittgenstein, learning involves mastering new rules, and similarly, a newcomer's integration into a community of practice can be understood as a gradual process of adopting and mastering these rules.

To illustrate these ideas, consider two scenarios involving the mathematical proposition “150 - 110 = 40”: i. a student is solving operation "150 - 110" using written algorithms; ii. a shopkeeper calculating change after receiving three $50.00 bills for a product costing $110.00. Although the proposition "150 - 110 = 40" remains universally true, the strategies employed differ depending on the community of practice. The student typically uses an algorithm, the most common school-taught solution. The shopkeeper, however, may use alternative methods, such as mental math, a calculator, or breaking the calculation into simpler steps (e.g., 150 - 100 = 50, then 50 - 10 = 40). Each community's approach reflects its distinct practices, and while these strategies can be evaluated for appropriateness, efficiency, or relevance, no value judgment can be made about their superiority. Both lead to the same conclusion: “150 - 110 = 40.” This highlights that mathematical propositions, while normative, are not *a priori* truths but emerge through shared practices.

Another compelling example involves a carpentry class in which a math teacher was asked to find the center of a circular piece of wood. The teacher used plane geometry to construct a circumscribed square and locate the circle’s center by the intersection of its diagonals. However, the carpenter solved the problem using a practical technique: he positioned the wood using a carpenter’s square and drew intersecting lines at 45°. Both methods were valid, but the carpenter's solution, rooted in specific tools and tacit knowledge, was more efficient for the practical context.
This scenario raises an important question: was the problem a mathematical one, or simply one that could be solved using mathematical methods? In practical settings like carpentry, problems cannot be separated from the tools and processes of the field. In contrast, a classroom exercise might reframe the problem as purely mathematical, requiring students to apply geometric principles without consideration for real-world tools or efficiency. These differences demonstrate how solutions depend on the context in which problems are situated.

This distinction highlights the importance of integrating diverse practices into mathematical education. The carpenter’s solution reflects one "language game," while the mathematician's reflects another, and both are embedded in their respective forms of life. As Gerrard (1987, p.99) observes, "Mathematics is a language game (…) its nature is best clarified by examining the role it assumes in our lives and its relationship to other language games." The challenge for education is to reconcile these forms of life, enabling students to navigate both practical and formal mathematical contexts. To bridge this gap, schools must create environments where practical and formal problem-solving coexist. Students should experience diverse strategies for mathematizing reality while also engaging with real-world practices from other communities. This approach can democratize access to mathematics, challenging its perceived superiority by demonstrating that multiple valid strategies exist for solving problems, each suited to specific contexts.

Conclusions

The synthesis of Wittgenstein’s discussions can be understood as an opposition between realist and pragmatic perspectives on the nature of mathematics and the production of meaning. In a realist conception, mathematical expressions serve purely semantic functions, with their meanings being independent of their uses. This view assumes a conventional relationship between signs and entities, whose existence or reality is established *a priori*. Language, in this context, is understood to have a primarily referential function, aligned with a Platonic perspective that entails "an absolutism of mathematical knowledge as a system of definitive, eternal truths that cannot be modified by human experience" (D'Amore, 2005, p.26). It is important to note that the idea of "modifying mathematics through experience" does not imply verifying or refuting its validity empirically. Instead, it repositions mathematical practice away from "discovery" and toward "invention."

From a realist standpoint, the analysis of mathematical expressions and objects is framed as "scientific" and logical, where knowledge is absolute and accessed through discovery. Pragmatism, by contrast, views the analysis of mathematical expressions and objects as subjective and non-absolute, with knowledge being contingent on circumstances and appropriated through use. Within a pragmatic framework, linguistic expressions acquire meaning based on their context of use, while mathematical objects take on different meanings depending on the problems and situations in which they are applied. Realist perspectives, however, often reinforce a Platonic ideal of language and mathematics. This view can influence teaching practices, perpetuating models that, at a broader level, contribute to discourses that distance access to mathematics from the majority of students.

From language games and situated learning perspectives, mathematics can also be understood as a social practice. The development of mathematical modeling, as well as the work of mathematicians or professionals who apply mathematics, involves elements such as mentorship, collaboration between experts and novices, and the use of specific procedures, languages, and objectives — similar to any other social practice. Lave (1996) also emphasizes that formal or school-based knowledge is not always easily applied to solving practical problems. For instance, when solving quantitative problems, people rarely arrive at wrong answers, since they have a clear notion of the meaning of the quantitative relationships they are looking for and of what a numerical solution will be in approximate terms. Even though they also have a very strong notion of the meaning of what they are doing, they are able to put aside the problems they recognize they are unable to solve within the time and for the reasons available.

While the concept of learning as legitimate peripheral participation provides valuable insights into teaching and learning processes, its implications for the classroom should be approached with caution. As Lave & Wenger (1991, p.40) explains, "legitimate peripheral participation is not in itself an educational form, much less a pedagogical strategy or a teaching technique. It is an analytical point of view on learning, a way of understanding learning."
As an analytical tool, one of its primary contributions is its critique of hegemonic learning theories, which often view learning as the straightforward result of cultural transmission. Such theories conceptualize learning as an "unproblematic unfolding, a result of transmission or socialization" (Lave, 2015, p.38). When combined with assumptions from cognitive psychology, this perspective frames learning as a "cognitive unfolding of teaching," reducing it to the school's role in transmitting culture (Lave, 2015) situated theories, therefore, challenge traditional assumptions by emphasizing the relational and contextual nature of learning, presenting new possibilities for understanding education beyond the confines of formalized teaching methods.

From a practical point of view, considering both everyday school life and initial teacher training, the discussions presented so far can be developed into various actions. The most immediate is that understanding mathematics as a human invention rather than a discovery brings it closer to individuals, making them feel empowered to learn it and, even more, to use it to solve everyday problems, construct arguments and justifications, that is, to express themselves mathematically. In this sense, since mathematics is a human creation, it is also diverse. It is a product of different cultures and different historical moments. Becoming aware of this fact helps to bring visibility to these different types of mathematics beyond the purely Eurocentric view of what mathematics is. This creates layers of representativeness for students to recognize their cultural backgrounds, those of their parents, and their ancestors.

This means bringing mathematics from different cultures into the classroom: the history of mathematics, different number systems, counting strategies, calculation procedures, geometric solutions, etc. However, it is essential to avoid the trap that many teachers anchored in Eurocentric models of mathematics and education fall into, which is to treat such knowledge only as curiosity or something exotic. It should be treated seriously, that is, as legitimate forms of mathematical knowledge. This implies not only that teachers should bring such knowledge into the classroom, but especially that they promote a space in which students and their families can bring their personal perspectives on mathematics. However, this type of situation depends on the creation of appropriate didactic conditions. One of these is to work, for example, with projects, mathematical investigations, or presenting problems with more than one solution or even no solution. This allows students to mobilize their knowledge (mathematical or non-mathematical) in search of a solution to the problem or to draw conclusions about the situation presented. Understanding mathematics from this perspective not only enables these types of activities, but also opens up space for a critical look at mathematics itself: why this definition or that one? What is behind certain algorithms or procedures? How have certain concepts changed throughout history? This type of questioning should serve not only for classroom discussions, but also as a research opportunity for teachers themselves.

Finally, this whole context reverberates in the field of educational assessment. It is not enough to recognize “other” mathematics as legitimate and accept only “one” curricular mathematics as an answer in tests and exams. In other words, not only do assessments need to be adapted, but so do the curricular structures themselves.
Considering that curriculum standards, teaching materials, educational systems, and assessments are still far from, or only slightly closer to, the perspective presented here is still a long way to go. However, it is through initiatives, albeit sporadic, by teachers in basic education or in initial training courses that we can glimpse the development of an educational culture that considers mathematics from other perspectives.

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